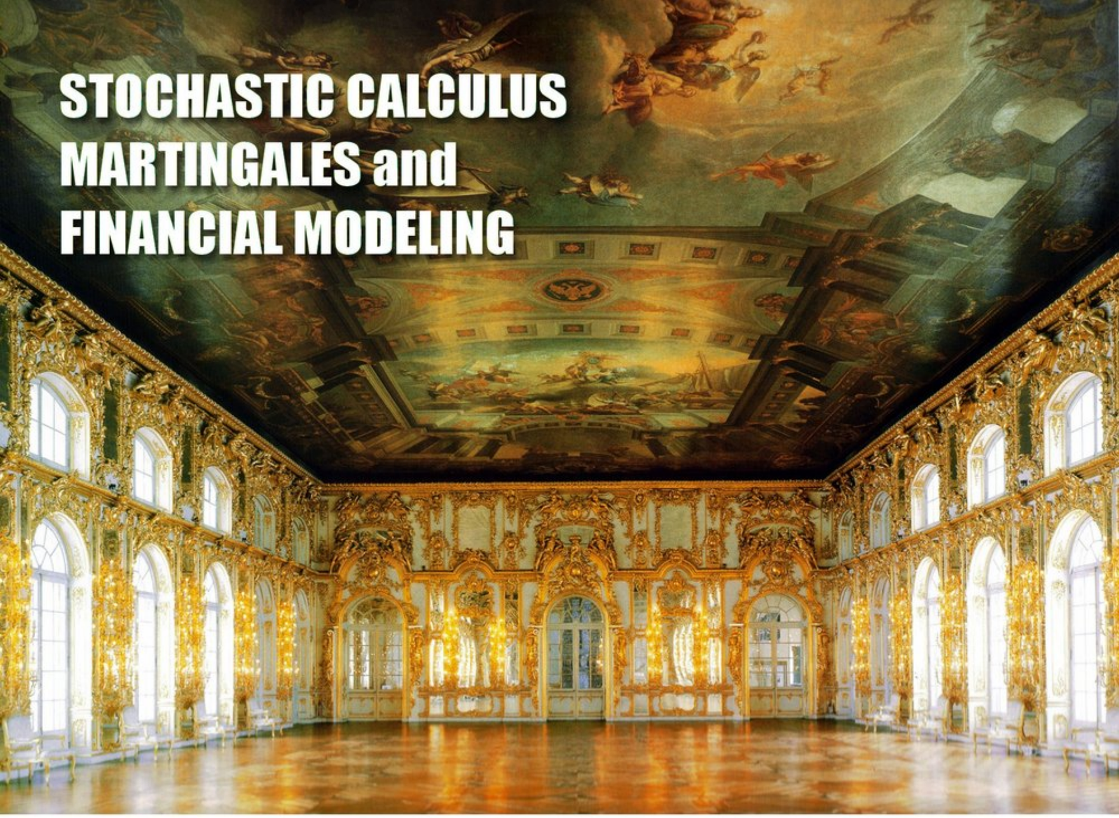

STOCHASTIC CALCULUS MARTINGALES and FINANCIAL MODELING

Book of abstracts

Saint Petersburg
June 30 – July 5, 2014

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Integrated option pricing models: an application of Fourier transforms

Akhlaque Ahmad

National Institute of Securities Markets, India

We start with Black Scholes model and analyze the impact of stochastic volatility, stochastic interest rate, Poisson Jumps and Levy Jumps on option prices. We use Square Root process and OU process to model the stochastic volatility and interest rates. Further, Poisson and Levy process is used to model the large & small jumps in stock prices respectively. Each stochastic factor is treated as a module and a family of integrated models is developed by integrating above stochastic factors using characteristic functions. We calculate option prices utilizing Inverse Fourier Transforms and observe significant correction in prices.

Optimal stopping under probability distortions

Denis Belomestny

Duisburg-Essen University, Germany

In this talk we consider the optimal stopping problems under probability distortions which include such well known risk measures as weighted AV@R. Our main result is a general minimax theorem which allows to solve the above optimal stopping problems via dynamic programming. Moreover, we address the question of the existence of the optimal stopping time and derive the corresponding additive dual representation.

Joint work with V. Kraetschmer.

Mathematical models for the formation of financial bubbles

Francesca Biagini

University of Munich, Germany

The notion of an asset price bubble has two ingredients. One is the observed market price of a given financial asset, the other is the asset's intrinsic value, and the bubble is defined as the difference between the two. The intrinsic value, also called the fundamental value of the asset, is usually defined as the expected sum of future discounted dividends. Here we study a flow in the space of equivalent martingale measures and focus on the corresponding shifting perception of the fundamental value of a given asset in an incomplete financial market model. This allows us to capture the birth of a perceived bubble and to describe it as an initial submartingale which then turns into a supermartingale before it falls back to its initial value zero. We illustrate our results in two examples, one due to Delbaen and Schachermayer [2] and the other given by a a variant of the stochastic volatility model discussed by Sin in [3]. This talk is based on the paper [1].

[1] Biagini F., Föllmer H., Nedelcu S. Shifting Martingale Measures and the Birth of a Bubble as a Submartingale, Finance and Stochastics, 18 (2), 297–326, 2014.

[2] F. Delbaen and W. Schachermayer. A simple counter-example to several problems in the theory of asset pricing. Mathematical Finance, 8, 1–12, 1998.

[3] C.A. Sin. Complications with stochastic volatility models. Advances in Applied Probability, 30(1), 256–268, 1998.

Martingale problem for path dependent diffusion processes with jumps

Jocelyne Bion-Nadal

Ecole Polytechnique, France

The martingale problem associated with a diffusion process with continuous coefficients has been introduced by Stroock and Varadhan [3], and then extended by Stroock [2] to the case of diffusion processes with Levy generators.

The subject of the present work [1] is to study the martingale problem associated with jump diffusions whose coefficients are path dependent. This means that one considers the operators

$$L^{a,b}(t,\omega) = \frac{1}{2}\sum_1^n a_{ij}(t,\omega)\frac{\partial^2}{\partial x_i \partial x_j} + \sum_1^n b_i(t,\omega)\frac{\partial}{\partial x_i}$$

$$K^M(t,\omega)(f)(x) = \int_{\mathbb{R}^n - \{0\}} \left[f(x+y) - f(x) - \frac{y^* \nabla f(x)}{1+||y||^2} \right] M(t,\omega,dy)$$

In [3] and [2], the functions $\phi = a, b, M$ depend at time t only on the value of the path at time t, $\phi(t,\omega) = \tilde{\phi}(t, X_t(\omega))$. Stroock has proved the existence and uniqueness of the solution to the martingale problem when the coefficients $\tilde{\phi}$ are continuous functions of (t,x) in $\mathbb{R}_+ \times \mathbb{R}^n$ (under the usual assumption of strict ellipticity for a).

In the present work, the functions a, b, M are defined on $\mathbb{R}_+ \times \Omega$ where Ω is the space $\mathcal{D}([0,\infty[, \mathbb{R}^n)$ of càdlàg paths with values in $\mathbb{R}^n$. The functions a, b, M are progressive which means that for given t, $a(t,\omega)$, $b(t,\omega)$ and $M(t,\omega)$ depend on the whole past of the trajectory ω up to time t.

The goal is to study the existence and uniqueness of the solution to the martingale problem. For $r \geq 0$ and $\omega_0 \in \Omega$, the probability measure $Q^{a,b,M}_{r,\omega_0}$ on the space Ω of càdlàg paths is solution to the path dependent martingale problem for $L^{a,b} + K^M$, starting from ω_0 at time r, if

$$Y^{a,b,M}_{r,t} = f(X_t) - f(X_r) - \int_r^t (L^{a,b}(u,\omega) + K^M(u,\omega))(f)(X_u)du$$

is a $(Q^{a,b,M}_{r,\omega_0},\ \mathcal{B}_t)$ martingale for every function f in $\mathcal{C}^\infty_0(I\!R^n)$ and if $Q^{a,b,M}_{r,\omega_0}(\{\omega \in \Omega\ |\omega_{|[0,r]} = \omega_{0|[0,r]}\}) = 1$, $\mathcal{B}_t$ being the canonical filtration.

One has first to choose the appropriate continuity properties for a, b, M. To have a Polish space, we choose the Skorhokod topology on Ω. Asking for continuity in (t, ω) would be too strong: even if g is continuous, the function $\tilde{g}(t, \omega) = g(t, X_t(\omega))$ is not continuous. This leads us to introduce a new approach for progressive functions. To every progressive function f defined on $I\!R_+ \times \Omega$ one associates a function $\overline{f}$ in three variables defined on $I\!R_+ \times \Omega \times I\!R^n$ such that $f(t, \omega) = \overline{f}(t, \omega, X_t(\omega))$. The required regularity properties for f are then the usual regularity properties for $\overline{f}$.

We prove the existence and the uniqueness of the solution to the martingale problem under the above continuity property for the coefficients, a, b, M, and the strong ellipticity of a. The proof of uniqueness is very challenging. It needs the introduction of a new norm on the space of progressive functions which serves as substitute for the $L^p(I\!R_+ \times I\!R^n)$ norm (in the usual case). The uniqueness result relies then on the proof of the invertibility of some linear operator in the Banach space constructed from the closure of progressive functions for this new norm.

A financial application of this result is the robust pricing/dynamic risk measuring under model uncertainty, that is when the possible law for the underlying asset is assumed to belong to a possibly non dominated set of probability measures on the set of paths.

[1] Bion-Nadal J., Path dependent diffusion processes with jumps, in preparation.

[2] Strook D., Diffusion processes asociated with Levy generators, Z. Wahrscheinlichkeitstheorie verw. Gebiete **32**, pp. 209-244 (1975).

[3] Stroock D. and Varadhan S., Diffusion processes with continuous coefficients, I and II, Communications on Pure and Applied Mathematics, **22**, pp 345-400 (1969).

An effective method to solve optimal stopping problems for Lévy processes in infinite horizon

Elena Boguslavskaya

Brunel University London, UK

We present a a new effective method to solve optimal stopping problems in infinite horizon for Lévy processes. The method can work for non-monotone reward functions and is especially straight forward for the reward functions which can be represented as polynomials, linear combinations of exponentials or linear combinations of exponential polynomials.

The method allows to avoid complicated differential or integro-differential equations which arise if the standard methodology is used.

To solve the problem we introduce two new objects. Firstly, we define a random variable $\eta(x)$ which corresponds to the $argmax$ of the reward function. Secondly, we propose a certain integral transform which can be built on any suitable random variable. It turns out that this integral transform constructed from $\eta(x)$ and applied to the reward function produces an easy and straightforward description of the optimal stopping rule. We illustrate our results with examples.

In the end we discuss the correspondence between the newly defined integral transform and the change of measure.

Entrepreneurial decisions on effort and project with a non-concave objective function

Abel Cadenillas

University of Alberta, Canada

We propose and solve a general entrepreneurial/managerial decision making problem. Instead of employing a specific class of utility functions, we use a general utility function. We approach the problem by a martingale method. We show that the optimization problem with the non-concave objective function has the same solution as the optimization problem when the objective function is replaced by its concave hull, and thus the problems are equivalent to each other. The value function is shown to be strictly concave and satisfies the Hamilton-Jacobi-Bellman equation of dynamic programming. We also show that the final wealth cannot take values in the region where the objective function is not concave: the entrepreneur would like to avoid ending up her/his wealth in the non-concave region. Because of this, her/his risk taking explodes as time nears maturity if her/his wealth is equal to the right end point of the non-concave region.

This is a joint work with A. Bensoussan and H.K. Koo.

Optimal discretization of hedging strategies with directional views

Jiatu Cai

University Paris-Diderot, France

We consider the hedging error of a derivative due to discrete trading in the presence of a drift in the dynamics of the underlying asset. We suppose that the trader wishes to find rebalancing times for the hedging portfolio which enable him to keep the discretization error small while taking advantage of market trends. Assuming that the portfolio is readjusted at high frequency, we introduce an asymptotic framework in order to derive optimal discretization strategies. More precisely, we formulate the optimization problem in terms of an asymptotic expectation-error criterion. In this setting, the optimal rebalancing times are given by the hitting times of two barriers whose values can be obtained by solving a linear-quadratic optimal control problem. In specific contexts such as in the Black-Scholes model, explicit expressions for the optimal rebalancing times can be derived.

Understanding stochastic volatility in financial markets (and the Tobin tax)

Albina Danilova

London School of Economics, UK

We develop a tractable model in which trade is generated by asymmetry in agents' information sets. We show that, even if news are not generated by a stochastic volatility process, in the presence of information treatment and/or order processing costs, the (unique) equilibrium price process is characterised by stochastic volatility. The intuition behind this result is simple. In the presence of trading costs and dynamic information, agents strategically chose their trading times. Since new information is released to the market only at trading times, the price process sampled at trading times is not characterised by stochastic volatility. But since trading and calendar times differ, the price process at calendar times is the time change of the price process at trading times – i.e. price movements on the calendar time scale are characterised by stochastic volatility. Our closed form solutions imply that: i) volatility is autocorrelated and is a non-linear function of both number and volume of trades; ii) the relative informativeness of numbers and volume of trades depends on the sampling frequency of the data; iii) volatility as well as liquidity, in terms of tightness, depth, and resilience, are jointly determined by information asymmetries and trading costs. The model is able to rationalise a large set of empirical evidence about stock market volatility, liquidity, and market frictions, and provides a natural laboratory for analysing the equilibrium effects of a financial transaction tax.

Robustness of quadratic hedging strategies in finance via Fourier transforms

Catherine Daveloose

Ghent University, Belgium

Asset price models based on Lévy processes, e.g. exponential Lévy, are well established in literature. Several methods are available to price and hedge related options. One of these methods is based on Fourier transforms. This leads to formulas for the option price and the Greeks in terms of the characteristic function of the driving Lévy process and of the Fourier transform of the payoff function for European vanilla options, see [3]. On the other hand since Lévy models imply incomplete markets, perfect hedging is impossible. However partial hedging can be reached via quadratic hedging strategies. For markets observed in a martingale, resp. semimartingale, setting the quadratic hedging strategies are computed by Fourier transform techniques in [5], resp. [4].

To compute the option price, the Greeks, or the position in the quadratic hedging strategies at any date before time of maturity, simulated prices of the underlying asset are required. However simulation of Lévy processes with infinite activity is hard. The approximation introduced in [1], based on replacing the jumps with absolute size smaller than s of a Lévy process by a scaled Brownian motion, facilitates this simulation issue. One could also look at this approximation from a modelling point of view, since one can choose to consider infinitely small variations coming either from a Brownian motion, or from a Lévy process with infinite activity. For s tending to zero, the approximation clearly converges in distribution to the original Lévy process. Even though convergence of asset prices does not necessarily imply the convergence of option prices, it was proved in [2] that for the considered models the related option prices and the deltas are robust. The question remained whether quadratic hedging is also robust.

We reconsidered the robustness conditions in [2] and proved the convergence of the quadratic hedging strategies. Moreover we computed convergence rates and motivated the applicability of our results with examples. In other words, it is justified to use the approximation and facilitate simulations with it.

This is a joint work with Asma Khedher and and Michéle Vanmaele.

[1] Asmussen, S., Rosinski, J.: Approximations of small jump Lévy processes with a view towards simulation. Journal of Applied Probability 38, 482–493 (2001)

[2] Benth, F., Di Nunno, G., Khedher, A.: Robustness of option prices and their deltas in markets modelled by jump-diffusions. Communications on Stochastic Analysis 5(2), 285–307 (2011)

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[4] Hubalek, F., Kallsen, J., Krawczyk, L.: Variance-optimal hedging for processes with stationary independent increments. Annals of Applied Probability 16, 853–885 (2006)

[5] Tankov, P.: Pricing and hedging in exponential Lévy models: review of recent results. Paris-Princeton Lecture Notes in Mathematical Finance (2010)

Monetary utility functions with convex level sets

Freddy Delbaen

ETH Zurich, Switzerland

Monetary utility functions are – except for the expected value – not of von Neumann-Morgenstern type. In case the utility function has convex level sets in the set of probability measures on the real line, we can give some characterisation that comes close to the vN-M form. For coherent utility functions this was solved by Ziegel. The general concave case under the extra assumptions of weak compactness was solved by Stephan Weber. In the general case the utility functions are only semi continuous. Using the fact that law determined utility functions are monotone with respect to convex ordering, we can overcome most of the technical problems. Having convex level sets can be seen as a weakened form of the independence axiom in the vN-M theorem.

Polymodels and portfolio construction under extreme risks

Raphael Douady

University Paris 1, France

A polymodel is a parsimonious estimate of a high-dimensional function (a hyper-surface) by using low-dimensional local approximations. A polymodel is described in terms of an algorithm but is nevertheless a function in its most standard sense as a description of a dependency between a dependent variable y and a set of k independent variables $x = (x_1, x_2, \ldots, x_k)$.

Its value is in that it takes into account the fact that functional relationships between the variables can differ in different domains of x, much like a regime switching model. However, a polymodel is a description of the dependency that can ab initio be specified with a rich structure and the estimation procedure will fit a parsimonious description for each potential domain of the problem. It does not require prior definition of domains, or prior specification of the dynamics within a domain. For example, if in a specific domain of x, the dependency between x and y can most parsimoniously be captured by a single factor linear model, the polymodel will capture this and exclude all other variables.

The objective of a polymodel is to efficiently estimate a function $F(x_1, x_2, \ldots, x_k)$ of many independent variables (k is large), that themselves can be related in complicated nonlinear ways, in particular with a view of correctly estimating the extreme behavior of the dependent variable with respect to the independent ones. In other words, the objective is to fit a high-dimensional hyper-surface which is given as a function $F(\cdot)$ of stochastic variables with a general joint distribution $P(x_1, x_2, \ldots, x_k)$. The key is that k is large enough that multivariable models are not easily estimated from the available data.

As noted, the polymodel approach can be useful in situations where the problem is highly multidimensional, however it can be also useful in situations where the variables upon which F depends are initially not known. In this case, the polymodel will in each domain of the independent variables, in a parsimonious way select the relevant variables which most contribute to the description of F. In this case, a polymodel can be thought of as a data mining tool useful in discovering hidden relationships between variables.

The estimation of a high-dimensional surface via a polymodel is achieved by locally fitting single- or multi-variable models (possibly nonlinear). These "elementary models" are of much smaller dimensionality then the full hyper-surface, and indeed in many cases a simple one variable function. The selection of independent variables that contribute most to the dependent variable is done parsimoniously. A polymodel will select the smallest possible model for a given domain. If however the complexity of the function calls for more variables, the polymodel is a rich enough structure to allow that.

Polymodels are in some ways similar to LOESS regression models, in which a nonlinear function is approximated locally by linear functions. A polymodel in a similar manner approximates a high-dimensional function locally with low-dimensional functions. This approach has proven its efficiency for estimating the extreme risks of financial assets and investments.

Valuation in illiquid markets and the Feynman-Kac representation

Ernst Eberlein

University of Freiburg, Germany

After a long period with an abundance of liquidity in the markets, the 2007 – 2009 financial crisis illustrated in a dramatic way how fundamental liquidity risk is. In this situation many securities with an excellent rating could no longer be traded. What is the value of the instruments under these market conditions? The classical valuation theory which is based on the law of one price assumes implicitly that market participants can trade freely in both directions at the same price. In the absence of perfect liquidity the law of one price should be replaced by a two price theory where the terms of trade depend on the direction of the trade.

We develop here a static as well as a continuous time theory for two price economies. The two prices are termed bid and ask or lower and upper price but they should not be confused with the vast literature relating bid-ask spreads to transaction costs or other frictions involved in modeling financial markets. The bid price arises as the infimum of test valuations given by certain market scenarios whereas the ask price is the supremum of such valuations. The two prices correspond to nonlinear expectation operators. The approach is made operational by using probability as well as measure distortions. We discuss in detail the validity of the Feynman-Kac representation of solutions of partial integro-differential equations on which the dynamic theory is based.

Specific models which are driven by purely discontinuous Lvy processes are considered. The approach is illustrated to price contracts with extremely long maturities. We also discuss the valuation of insurance loss liabilities modeled via compound Poisson processes.

This is a joint project with D. Madan, M. Pistorius, W. Schoutens and M. Yor.

[1] Eberlein, E., Madan, D., Pistorius, M., Schoutens, W., and Yor, M.: Two price economies in continuous time. Annals of Finance 10 (2014), 71-100

[2] Eberlein, E., Madan, D., Pistorius, M., and Yor, M.: Bid and ask prices as non-linear continuous time G-expectations based on distortions. Mathematics and Financial Economics (2014) (to appear)

[3] Eberlein, E., Glau, K.: Variational solutions of the pricing PIDEs for European options in Lvy models. Applied Mathematical Finance (2014) (to appear)

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On the chaotic representation property of certain families of martingales

Hans-Jürgen Engelbert

Friedrich Schiller-University, Jena, Germany

Let $T > 0$ be a finite time horizon, $(\Omega, \mathscr{F}, \mathbf{P})$ a complete probability space and $\mathbb{F} = (\mathscr{F}_t)_{t\in[0,T]}$ a filtration satisfying the usual conditions. By $\mathscr{H}^2 = \mathscr{H}^2(\mathbb{F})$ we denote the Hilbert space of square integrable martingales X on $[0,T]$ with respect to $\mathbb{F}$. In the present talk we consider families $\mathscr{X}$ of martingales contained in $\mathscr{H}^2$. We assume that for every $X, Y \in \mathscr{X}$ the predictable covariation $\langle X, Y\rangle$ is deterministic. We can therefore introduce, for every $n \in \mathbb{N}$, the space $\mathscr{J}_n(T)$ of terminal values of n-fold iterated stochastic integrals generated by the family $\mathscr{X}$. The spaces $\mathscr{J}_n(T)$ and $\mathscr{J}_m(T)$ are orthogonal in $L^2(\Omega, \mathscr{F}, \mathbb{P})$ for every $n \neq m$. We denote by $\mathscr{J}(T)$ the closure in $L^2(\Omega, \mathscr{F}^{\mathscr{X}}, \mathbb{P})$ of the linear hul! l of $\mathbb{R} \cup \bigcup_{n=1}^{\infty} \mathscr{J}_n(T)$. We say that $\mathscr{X}$ possesses the *chaotic representation property* (CRP) if $\mathscr{J}(T) = L^2(\Omega, \mathscr{F}^{\mathscr{X}}, \mathbb{P})$. If the family $\mathscr{X}$ is orthogonal then the CRP amounts to saying that we have the orthogonal decomposition

$$L^2(\Omega, \mathscr{F}^{\mathscr{X}}, \mathbb{P}) = \mathbb{R} \oplus \bigoplus_{n=1}^{\infty} \mathscr{J}_n(T)\,. \tag{1}$$

If $\mathscr{X} = \{W\}$, W Brownian motion, this is just the chaos expansion of Itô (1951).

We call $\mathscr{X}$ compensated-covariation stable if for all martingales $X, Y \in \mathscr{X}$ the martingale $([X,Y] - \langle X, Y\rangle, \mathbb{F})$ again belongs to $\mathscr{X}$ where $[X,Y]$ denotes the covariation process of X and Y.

The main result of the present talk consists of the following

Theorem 1. *Suppose that the following conditions are satisfied:*

1) *The polynomials generated by* $\{X_t : \ t \geq 0, \ X \in \mathscr{X}\}$ *belong to* $L^2(\Omega, \mathscr{F}^{\mathscr{X}}, \mathbb{P})$ *and are dense.*

2) $\mathscr{X}$ *is compensated-covariation stable.*

3) $\langle X, Y\rangle$ *is deterministic for all* $X, Y \in \mathscr{X}$.

Then $\mathscr{X}$ *possesses the CRP.*

Under the conditions of Theorem 1, as a consequence of the CRP, the predictable representation property (PRP) of $\mathscr{X}$ with respect to the filtration $\mathbb{F}^{\mathscr{X}}$ is satisfied, too.

As an important application, we shall construct families $\mathscr{X}$ of martingales related with Lévy processes L satisfying the CRP with respect to $\mathbb{F}^L$. The chaos decomposition (1) gives another representation of Itô's chaos expansion in terms of *multiple* Itô integrals introduced by Itô (1956). These results also include the CRP for the orthogonalized Teugels martingales which was studied by Nualart and Schoutens (2000).

This talk is based on joint work with Paolo Di Tella (Humboldt University of Berlin). Work supported in part by the European Community's FP 7 Programme under contract PITN-GA-2008-213841, Marie Curie ITN "Controlled Systems".

Spatial risk measures: local specification, aggregation, and phase transition

Hans Föllmer

Humboldt University of Berlin, Germany

The quantification of financial risk in terms of convex risk measures is closely related to the microeconomic theory of preferences in the face of risk and Knightian uncertainty. We discuss some of these connections and then turn to the implications of dynamic or spatial consistency for convex risk measures on product spaces.

In the spatial setting of a large network, the local specification of convex risk measures can be seen as a non-linear extension of the local specification of equilibrium states in Statistical Mechanics. We discuss the corresponding aggregation problem of passing from local to global risk measures and the appearance of phase transitions. This will involve a non-linear extension of backwards martingale convergence, arguments from preference theory, and a spatial version of Dynkin's construction of the entrance boundary of a Markov process.

On the limit behavior of option hedging sets under transaction costs

Julien Grépat

CEREMADE, University of Paris–Dauphine, France

Though continuous trading is a part of the standard paradigm of modern finance, in practice, usually, portfolio revisions are done along a discrete-time greed. In the case of proportional transaction costs the agents know the order of total number of transactions and agree between them on a transaction costs coefficient. For more frequent revisions one can expect a smaller level of the latter.

The aim of this presentation is to study the convergence of super-replication sets for a European option in a sequence of discrete–time models with time step T/n and evanescent transaction costs. In the multi–asset framework presented in [1], each model of a sequence generates a set of hedging endowments and the problem is to find a limit for these sets in the sense of closed topology. When the transaction costs decrease at rate $n^{-1/2}$, we give a description of the limit set for specific models. It happens to be different from that of the limiting continuous–time model based on geometric Brownian motions. We deduce inclusions for general models.

[1] Kabanov, Y. and Safarian, M. *Markets with transaction costs*, Springer Finance, Springer, Berlin, 2009.

On portfolio optimization and indifference pricing with small transaction costs: rigorous proofs based on duality

Jan Kallsen

Kiel University, Germany

Portfolio optimization problems with frictions as e.g. transaction costs are hard to solve explicitly. In the limit of small friction, solutions are often of much simpler structure. In the last twenty years, considerable progress has been made both in order to derive formal asymptotics as well as rigorous proofs. However, the latter usually rely on rather strong regularity conditions, which are hard to verify in concrete models. Some effort is still needed to make the results really applicable in practice. This talk is about a step in this direction. More specifically, we discuss portfolio optimization for exponential utility under small proportional transaction costs. As an example, we reconsider the Whalley-Willmott results of utility-based pricing and hedging in the Black-Scholes model. We relax the conditions required by Bichuch who gave a rigorous proof for smooth payoffs under sufficiently small risk aversion.

Equilibrium in risk sharing games

Konstantinos Kardaras

London School of Economics, UK

We study equilibrium sharing of investment risk among agents whose random endowments constitute private information. Given the sharing rules that optimally allocate the submitted endowments, we propose a Nash game where agents' strategic choices consist of the endowments to be submitted for sharing. It is proved that the best response problem admits a unique solution (which we call "best endowment response") and differs from the agent's true risk exposure. Then, we proceed in showing that the Nash equilibrium risk sharing admits a finite dimensional characterisation, and that it exists and is unique in the case of two agents. Analysis shows that the game benefits the agents close to risk neutrality, since their expected utilities are higher at the Nash risk sharing equilibrium than the optimal risk-sharing one.

Joint work with Michail Anthropelos.

A unified approach for the pricing of generalized Asian option

Masaaki Kijima

Tokyo Metropolitan University, Japan

In this paper, we consider a generalized Asian option that includes any type of Asian options studies in the literature as a special case, and propose a unified approximation method for the pricing of generalized Asian options when the underlying process is a diffusion. Through ample numerical examples, we show that the accuracy of our approximation remains quite high even for very complex Asian options with long maturity and high volatility. Comparisons are made with the existing methods in the literature. Our method can be easily extended to the case of stochastic volatility and some numerical results are reported.

Joint work with H. Funahashi.

A mutlti-dimensional quadratic BSDE arising in a price impact model

Dmitry Kramkov

Carnegie Mellon University, USA

We consider an inverse problem to optimal investment which originates in the price impact model of Grossman and Miller (1988). We are given an exponential utility function $U(x) = \frac{1}{a}e^{-ax}$, $x \in \mathbf{R}$, with the absolute risk-aversion $a > 0$, the stocks dividends Ψ, and the predictable process γ of the number of stocks. We have to determine the stock prices S with the terminal value $S_T = \Psi$ such that γ is the optimal investment strategy in the S-market. We show that the prices S are components of the solution to a multi-dimensional quadratic BSDE. We obtain that for small values of the risk-aversion a the solution to this BSDE is well-defined, that is, it exists and is (globally) unique. We construct an example showing that in general, even for bounded Ψ, the solution may not be well-defined.

The presentation is based on a joint work with Sergio Pulido from École Polytechnique Fédérale de Lausanne.

Bidding games and random walks of stock market prices

Victoria Kreps, Victor Domansky

St. Petersburg Institute for Economics and Mathematics, Russia

We consider multistage bidding models where m types of risky assets (shares) are traded between two agents that have different information on the liquidation prices of traded assets. The prices are random integer variables that are determined by the initial chance move according to a probability distribution $\mathbf{p}$ over the m-dimensional integer lattice that is known to both players. Player 1 is informed on the prices of all types of shares, but Player 2 is not. The bids may take any integer values. The model of n-stage bidding is reduced to a zero-sum repeated game with lack of information on one side. We show that, if liquidation prices of shares have finite variances, then the sequence of values of n-step games is bounded. This makes it reasonable to consider the bidding of unlimited duration that is reduced to the infinite game. We give the solutions for these games. The optimal strategy of Player 1 generates a random walk of transaction prices. But unlike the case of one-type assets, the symmetry of this random walk is broken at the final stages of the game.

Approximation of the solution of the backward stochastic differential equation. Small noise, ergodic diffusion and unknown volatility cases

Yury A. Kutoyants

University of Maine, France

We consider three problems of the approximation of the solution of the backward stochastic differential equation (BSDE) in the markovian case. We suppose that the trend coefficient or diffusion coefficient depend on some unknown parameter and we propose the approximations of the BSDE with the help of the one-step MLE of this parameter. The estimators are easy to calculate and the proposed approximations are asymptotically efficient in mean-square sense. We study three models of observations of the forward stochastic differential equation. The first one corresponds to the situation where the time of observation is fixed and the diffusion coefficient tends to zero (small noise). The second is devoted to the ergodic diffusion process and large samples approach. In the third problem the forward equation has unknown parameter in the volatility. In the first two problems the observations are in continuous time and in the third problem we suppose that the observations are in discrete times and we have the high frequency asymptotics.

Financial market models with friction: a general approach

Emmanuel Lepinette

Paris Dauphine University, France

A general financial market model is defined by a liquidation value process. This approach generalizes the conic models of Schachermayer and Kabanov where the transaction costs are proportional to the exchanged volumes of traded assets. This allows to consider financial market models where proportional transaction costs and fixed costs coexist. In this case, the solvency set of all portfolio positions that can be liquidated without any debt is not necessary convex. Therefore, the usual duality principle based on the Hahn-Banach separation theorem is not appropriate to characterize the prices super hedging a contingent claim. An alternative method is proposed to price European or American contingent claims under absence of arbitrage opportunity of the second kind.

A new approach to stochastic equations with irregular drift coefficients

Thilo Meyer-Brandis

University of Munich, Germany

We present a new method for the construction and study of solutions of stochastic equations with irregular drift coefficients. We first focus on demonstrating the principles of our techniques by analyzing strong solutions of finite-dimensional stochastic differential equations (SDE's) driven by Brownian motion. Strong solutions of SDE's with non-Lipschitz drift coefficients appear in various important applications e.g. in physics or stochastic control theory. Contrary to most approaches in the literature on SDE's with irregular coefficients, our method does not employ a pathwise uniqueness argument (or the Yamada-Watanabe theorem) but relies on a compactness criterion based on Malliavin calculus together with an approximation argument for certain generalized processes in the Hida distribution space which we directly verify to be strong solutions. An important consequence of our method is that the constructed strong solutions, which include the solutions derived by A. Y. Veretennikov for bounded and merely measurable drift coefficients, are Malliavin differentiable. Further, a major strength of our approach is that it can be transferred and employed to the analysis of different important aspects of solutions of various other types of stochastic equations with irregular coefficients besides finite-dimensional SDE's. In the second part of the talk we give a synopsis of further applications such as stochastic flows of Sobolev diffeomorphisms, Bismut-Elworthy-Li type formulas (which have applications in the computation of Greeks in Mathematical Finance), small noise considerations, and the study of solutions of certain classes of jump SDE's, resp. stochastic partial differential equations, resp. equations driven by infinite-dimensional Brownian motion.

Optimal time to sell a stock in the presence of gap, default and volatility risks

Aleksandar Mijatovic

Imperial College, London, UK

We consider a small investor who holds a stock that is subject to default-risk and seeks to identity the optimal time to sell the stock in the sense of minimizing the "prophet's drawdown" (i.e. the ratio of the ultimate maximum on a given horizon and the value of the stock price at the moment of sale). We assume that default occurs at a constant rate and that at the moment of default there is a random recovery value. We phrase this problem as a stochastic optimisation problem, which we solve explicitly in the case that the stock price before default is modelled by a spectrally negative exponential Lévy process. One of the insights provided by this solution is a natural additive decomposition of the optimal level of drawdown (at which the sale should take place) into contributions from gap, default and volatility risks.

This is joint work with Martijn Pistorius.

On the strong law of large numbers for some self-normalized stochastic processes

Ekaterina Palamarchuk

The Higher School of Economics, Russia

We establish the strong law of large numbers for some self-normalized stochastic processes. The numerator is a functional defined on solutions of linear stochastic differential equations with exponentially stable matrices. Several examples are provided. The results can be applied to study the long-run behavior of stochastic volatility and integrated stochastic volatility processes.

SDE, BSDE and path-dependent PDE

Shige Peng

Shandong University, China

Using G-expectation, we introduce a new derivative of path function under a generalized Sobolev spaces which lead a new 1-1 correspondence of the path-derivatives. A new form of Ito's formula for functions of Brownian path have been naturally and rigorously derived. This permit us to provide a 1-1 correspondence between a of backward SDEs driven by G-Brownian motion and a type of fully nonlinear path dependent PDEs. Many interesting and surprising examples are given to demonstrate this new tool of stochastic analysis. We also explain how to apply the above results to measure and regulate financial risks under probability model uncertainty.

On market models that satisfy no-arbitrage conditions weaker than NFLVR

Wolfgang Runggaldier

University of Padua, Italy

The modern theory of asset pricing relies heavily on the no-arbitrage condition given by NFLVR. On the other hand, real markets admit some weaker forms of arbitrage and so the interest arises in finding market models that satisfy no-arbitrage conditions weaker than NFLVR and nevertheless admit a satisfactory solution to the fundamental problems of stochastic finance. We discuss examples of (continuous and discontinuous) market models that satisfy NA1 (NUPBR) but not NFLVR.

Construction of models of a market intrinsic time based on limits of sums of processes of the pseudo-Poisson type processes

Oleg Rusakov

Saint-Petersburg State University, Russia

Pseudo-Poisson process (W. Feller II, X) in a wide sense is defined as a poissonian subordinator for a sequence of random variables. At every point of jump of the leading Poisson process, the corresponding current term of a subordinate sequence is replacing with the next term. The consecutive replacements of the terms of the subordinate sequence we interpret as realizations of events of intrinsic time of financial market. We consider the case of the stationary subordinate sequence and examine sums of independent copies of poissonian subordinators for such sequences when the intensity parameter of the leading Poisson process we permit to be a random one. In the case of the non-random intensity, the limit of the sums belongs to a class of the Ornstein-Uhlenbeck type processes. In the case of the random intensity, the dependence structure of the limit process is described by the Laplace transform of the intensity distribution.

This is a joint work with D. Apleev.

Integral representation of martingales motivated by the problem of market completion with derivative securities

Daniel C. Schwarz

Carnegie Mellon University, USA

A model of a financial market is complete if any payoff can be obtained as the terminal value of a self-financing trading strategy. It is well known that numerous models, for example stochastic volatility models, are however incomplete. We present conditions, which, in a general diffusion framework, guarantee that in such cases the market of primitive assets enlarged with an appropriate number of traded derivative contracts is complete. From a purely mathematical point of view we prove the following in- tegral representation theorem: let $\mathbb{Q}$ and $\mathbb{P}$ be two equivalent probability measures, S^F a d_F-dimensional martingale under $\mathbb{Q}$, ψ a d_B-dimensional vector of random variables, such that S^F and ψ are defined in terms of the solution X to a d-dimensional stochastic differential equation and let $S_t^B = \mathbb{E}^{\mathbb{Q}}[\psi|\mathcal{F}_t]$. We present conditions which guarantee that every local martingale under $\mathbb{Q}$ can be represented as a stochastic integral with respect to the d-dimensional martingale $S \triangleq (S^F \ S^B)^\star$. Notably, if $\psi \triangleq f(X_1)$, our conditions allow the matrix function $(\partial_{x^j} f^i)_{i=1,\ldots,d_B, j=d_F+1,\ldots,d}$ to be singular everywhere on $\mathbb{R}^d$ and are hence applicable to the verification of the completeness of markets in which, in addition to stocks, one may also trade derivative securities.

A new approach to bubbles in financial markets

Martin Schweizer

ETH Zürich, Switzerland

We introduce a new notion of bubbly markets. This is an economically motivated and formulated concept which captures the idea that a given financial markets contains "bubbles". We give dual characterisations of bubbly markets in terms of numeraires and martingale measures, and we show in particular that in any "reasonable" bubbly market, discounted prices must be strict local martingales, for any numeraire and any martingale measure associated to that numeraire. This can be viewed as a "robust" notion of a market containing a bubble. We show by examples how different concepts in our setting are related, discuss some of the related literature, and explain how existing work fits into our framework. If time permits, we also discuss the issue of valuation principles for financial products in such markets with bubbles.

The talk is based on ongoing joint work with Martin Herdegen (ETH Zürich).

Limit theorems for mixed stochastic differential equations with delay

Georgiy Shevchenko

Taras Shevchenko National University of Kyiv, Ukraine

The main object of the talk will be so-called mixed delay stochastic differential equation in $\mathbb{R}^d$:

$$X(t) = X_0 + \int_0^t a(X_s)ds + \int_0^t b(X_s)dW(s) + \int_0^t c(X_s)dB^H(s), \quad (1)$$

where $W = \{W(t), t \geq 0\}$ is a standard Wiener process, $B = \{B^H(t), t \geq 0\}$ is a fractional Brownian motion with the Hurst parameter $H > 1/2$. The coefficients of this equation depend on the past $X_s = \{X(s+u), u \in [-r, 0]\}$ of the process X, where $r > 0$ is a fixed delay horizon.

The motivation to consider such equations comes from financial mathematics, where it is useful to distinguish between two main sources of this randomness. The first source is the stock exchange itself with thousands of agents; the noise coming from this source can be assumed white and is best modeled by a Wiener process. The second source is the financial and economical background. The random noise coming from this source usually has a long range dependence property, which can be modeled by a fractional Brownian motion B^H with the Hurst parameter $H > 1/2$.

The talk will concentrate on limit theorems for equations (1), including stability of solutions under convergence of noises and coefficients. As a by-product, we will obtain convergence of Euler approximations and convergence of solutions to equations with vanishing delay.

Investment, collateral, and financing constraints

Takashi Shibata

Tokyo Metropolitan University, Japan

This paper examines the optimal investment timing decision problem of a firm constrained to a debt issuance limit determined by collateral value. We show that the investment thresholds have a U-shaped relation with the debt issuance limit constraints, in that they are increasing (decreasing) with the constraint for high (low) debt issuance limit. Debt issuance limit constraints lead to debt holders experiencing low risk and low returns. That is, the more severe the debt issuance limits, the lower the credit spreads and default probabilities. Our theoretical results are consistent with stylized facts and empirical results.

This is a joint work with Michi Nishihara.

Statistical sequential analysis of fractional Brownian motion

Albert Shiryaev

Steklov Mathematical Institute, Russia

We present several new results in sequential statistical analysis of fractional Brownian motion. It is well-known that this process is neither a Markov process nor a martingale (except the case when it is a standard Brownian motion), which makes standard sequential procedures inapplicable to it since they use, in one way or another, the martingale property or the Markov property of observable processes.

We consider the problem of sequential estimation of the drift of a fractional Brownian motion and the problem of testing hypotheses about its value. In our recent papers, we obtained explicit optimal decision rules in these problems. The solutions are based on the representation of the filtration of the observable process through a certain martingale connected with the fundamental martingale of Molchan and Golosov.

This is a joint work with M. Zhitlukhin and A. Muravlev.

Shifted small deviations for Lévy processes

Elena Shmileva

Saint Petersburg State University, Russia

In this talk we present results on the shifted small deviations for Lévy processes from the Wiener domain of attraction and for the symmetric stable Lévy processes. We also show how these results allow to obtain the Onsager-Machlup functional, optimization of which leads to the most probable sample paths of the processes.

Optimal stopping of Markov chains and related problems

Isaac Sonin

University of North Carolina at Charlotte, USA

The optimal stopping (OS) of a Markov chain (MC) is a classical problem of stochastic control. Its solution, at least on a principal level, can be found e.g. in the classical texts of Shiryayev and Chow, Robbins and Sigmund, or the more recent book of Peskir and Shiryaev. Since most practical problems of optimal stopping in mathematical finance or mathematical statistics usually are given on a finite time interval or in a nonmarkovian form, the attention of researchers shifted to specific problems, and to the development of specific methods for specific problems. It is worthwhile to recall the words of T. Ferguson: "Most problems of optimal stopping without some form of Markovian structure are essentially untractable". The main goal of this talk is to discuss the relationship between the Elimination Algorithm of OS of MC developed earlier by the author and the calculation of a generalized Gittins index for MC, (Whittle and Kathehakis & Veinott indices), and to some other problems of stochastic control, e.g. the recursive solution of the discrete versions of the Poisson and Bellman equations.

A quantitative method for the pricing of contingent claim under uncertainty

Xianming Sun

Central South University, Changsha, Hunan, China

Ghent University, Belgium

Risk analysis under model uncertainty and parameter uncertainty is attracting more and more public attention, especially after the 2008 crisis. Facing uncertainty at both levels of model and parameter, it is of great importance to calculate the conservative bid and ask prices of the OTC traded derivatives. Since the OTC derivatives can be partly hedged with liquid derivatives, the price of an OTC derivative should reflect the unhedgeable risk.

To price the target contingent claim under uncertainty, this paper proposes a quantitative method, which is based on the acceptable unhedgeable risk. The acceptability is characterized by assigning different weights to the candidate prices calculated with a specific type of the candidate models and different sets of the model parameters. The acceptability-based methodology mainly involves two issues:

1. how to accurately and efficiently calculate an ensemble of the target derivative prices with a set of pricing models, whose parameters are specified in the interval form,
2. how to assign subjective weights on an ensemble of the target derivative prices.

We address the aforementioned two issues by using the collocation method and a distortion function, respectively. Then, we can specify the bid and ask prices of the target derivative. The numerical results highlight the accuracy and efficiency of the collocation method used for the first issue, and the effects of the choice of different distortion functions in the second issue.

This is a joint work with Michéle Vanmaele.

Optimal capital injection problem under financial crisis

Teruyoshi Suzuki

Hokkaido University, Sapporo, Japan

We will introduce an optimal capital injection problem and propose an algorithm to solve it. The problem can be represented by a linear programming formulation under Eisenberg and Noes model. We present a sequential method to solve both the firms payoff to debt holders and the governments capital injection to the firms. We will show that the priority rule of capital injection does not depend on the amount of the budget by the government.

This is a joint work with Toshimasa Ishii and Jianming Shi.

Hedging under multiple risk constraints

Peter Tankov

University Paris-Diderot, France

Motivated by the asset-liability management of a nuclear power plant operator, we consider the problem of finding the least expensive portfolio which outperforms a given set of stochastic benchmarks at a sequence of future dates. For a specified loss function, the shortfall with respect to each of the benchmarks weighted by this loss function must remain bounded in expectation by a given threshold. We consider different risk constraints and alternative formulations of this problem in a complete market setting, and establish the relationship between these formulations. By using a recursive dynamic programming method, we solve the hedging problems in a general non-Markovian context and give explicit solutions in special cases. We finally present applications to an actual asset-liability management problem of an energy company in a realistic setting.

Polynomial interest rate and volatility models

Mike Tehranchi

Cambridge University, UK

We explore a class of interest rate and volatility models in which the prices of zero-coupon bonds and power options can be expressed as a polynomial of a state process. These models can be considered tractable generalisations of popular models such as those of Cox-Ingersoll-Ross, Heston and Stein-Stein. In particular, we characterise the family of diffusions which admit this type of tractability, in the spirit of Filipovic's classification of exponential polynomial term structure models. For instance, in the case of interest rates, suppose Z is a factor process verifying the SDE $dZ = b(Z)dt + \sigma(Z)dW$ and the spot interest rate is given by $r_t = R(Z_t)$. There exist linearly independent functions $g_0, \ldots, g_n$ such that the bond price is of the form

$$P(t,T) = \sum_{i=1}^{n} g_i(T-t)Z_t^i$$

only if R is quadratic, b is cubic, and σ^2 is quartic.

Joint work with Si Cheng.

A convergence theorem in the Emery topology and another view on the Fundamental Theorem of Asset Pricing

Josef Teichmann

ETH Zurich, Switzerland

We work in the general setting of admissible portfolio wealth processes as introduced, e.g., by Y. Kabanov (1997). We show that in this setting No unbounded Profit with bounded risk (NUPBR) implies the so called (P-UT) property, a boundedness property in the Emery topology. Combining this insight with well known results from Mémin and Słominski (1991) leads to a short variant of the proof of the fundamental theorem of asset pricing initially proved by Delbaen and Schachermayer. We also show a new result on Large Financial markets: (NAFL) can be replaced by an economically more convincing property reminiscent of No Free Lunch with vanishing risk but allowing fort the same conclusion.

Joint work with Christa Cuchiero and Irene Klein.

Skorokhod embedding and approximating diffusions with Markov chains

Mikhail Urusov

Duisburg-Essen University, Germany

In this talk we suggest a new method for approximating one-dimensional diffusions with Markov chains. The input is a driftless diffusion

$$dM_t = \eta(M_t)\, dW_t, \quad M_0 = m,$$

and a centred probability measure μ on $\mathbb{R}$. The output is a Markov chain such that the conditional distribution of its increments is an appropriate state-dependent rescaling of the measure μ, which approximates M. In the case when M is a Brownian motion the approximating Markov chains are random walks with the increments distributed according to μ and the scaling as in the Donsker-Prokhorov invariance principle. In general, this approximation is different from Stone (1963). The method of the proof is based on the extension of Ankirchner, Hobson and Strack (2013) to one-dimensional diffusions of the Bass solution of the Skorokhod embedding problem.

This is a joint work with Stefan Ankirchner and Thomas Kruse.

Indifference pricing for exponential Lévy models and HARA utilities

Lioudmila Vostrikova

University of Angers, France

Exponential Levy models are used in Mathematical Finance since long time. The popularity of such models can be explained firstly by their good feet to real data for one dimensional distributions of risky assets, and secondly, by their tractability from mathematical point of view.

To describe our risky assets, we suppose that we have two independent Levy processes $X^{(1)} = (X_t^{(1)})_{0\leq t\leq T}$ and $X^{(2)} = (X_t^{(2)})_{0\leq t\leq T'}$ with generating triplets (b_1, c_1, ν_1) and (b_2, c_2, ν_2) respectively. Let $X = (X_t)_{0\leq t\leq T}$ be Levy process obtained by linear combination of the processes of $X^{(1)}$ and $X^{(2)}$ with coefficients ρ_1 and ρ_2:

$$X_t = \rho_1 X_t^{(1)} + \rho_2 X_t^{(2)}$$

In such setting, our risky assets will be $S^{(1)} = (\mathcal{E}(X_t))_{0\leq t\leq T}$ and $S^{(2)} = (\mathcal{E}(X_t^{(2)}))_{0\leq t\leq T'}$ where $\mathcal{E}$ is Dolan-Dade exponential and $T' > T$.

We suppose that the agent who has risky asset $S^{(1)}$ and European type option $G(X_{T'}^{(2)})$ with some non-negative measurable function G, can hedge the asset $S^{(1)}$ but he cannot hedge $S^{(2)}$ because of legal restrictions or lack of liquidity, and he is interested by indifference price of the option $G(X_{T'}^{(2)})$.

Let us denote by $U(x, G)$ maximal expected utility with option:

$$U(x, G) = \sup_{\phi\in\Pi} \mathbb{E}u(x + \int_0^T \phi_s dS_s^{(1)} + G(X_{T'}^{(2)}))$$

where x is initial capital, u is utility function and Π is the set of admissible(asymptotically admissible) self-financing strategies. Then indifference prices p_b and p_s for buying and selling of the option verify:

$$U(x - p_b, G) = U(x, 0)$$

$$U(x + p_s, -G) = U(x, 0)$$

Indifference price for options was studied in many papers and books but essentially for the continuous semi-martingales models. In Ellanskaya, Vostrikova (2013) we obtained the equations and the formulas for indifference price in general case when risky assets are exponential semi-martingales with jumps. We will show how one can apply these results in the case of exponential Levy models with dependent Levy processes, and obtain mathematically and numerically tractable results.

Space-time structure of branching random walks

Elena Yarovaya

Moscow State University, Russia

Symmetric continuous-time branching random walks on d-dimensional lattices with a finite set of branching sources are considered. The problem on the spectrum of a bounded symmetric operator with multi-point potential generating a branching random walk plays an important role in the theory of branching random walks. Resolvent analysis of such operators has allowed to investigate branching random walks with large deviation. A special attention is paid to the case when the spectrum of the evolution operator of mean numbers of particles contains only one positive isolated eigenvalue. The limit theorems on asymptotic behavior of the Green function for transition probabilities were established for random walks with a finite variance of jumps. The obtained results allow to study the front of branching random walk and the structure of the particle population inside of the front and near to its boundary.

This work was partly supported by RFBR grant 13-01-00653. Joint work with S. Molchanov.

Invariance principle for variable speed random walks on trees

Anita Winter

Duisburg-Essen University, Germany

One of the most classic results in probability theory and statistics is the functional central limit theorem which states that suitably rescaled paths of random walks converge weakly towards the paths of Brownian motion. Around fifty years ago Stone generalized this to a class of R-valued strong Markov processes which have in common that their state space is a closed subset of R and that their paths "do not jump over points". If such a process has a discrete state space, then it is a random walk. If the state space is an interval, the process has continuous paths. When put in their "natural scale" these processes are determined by their so-called speed measure. Stone shows that the processes depend continuously on the speed measure.

In this talk we want to extend this result from R-valued Markov processes to Markov processes which take values in tree-like metric spaces. We will establish a one-to-one correspondence between metric measure trees (T, r, ν) and strong Markov processes $X = (X_t)_{t \geq 0}$ with values in the tree (T, r) whose speed measure can be identified as ν. We will show that a family of ν_n-speed motions on (T_n, r_n) converges in path space to the ν-speed motion provided that the underlying metric measure spaces converge in the Gromov-Hausdorff-vague topology. The topology will be introduced during the talk as well.

We will relate our invariance principle to several examples from the literature.

This is joint work with Siva Athreya (ISI Bangalore) and Wolfgang Löhr (Universität Duisburg-Essen).

Buy-low and sell-high investment strategies

Mihail Zervos

London School of Economics, UK

Buy-low and sell-high investment strategies are a recurrent theme in the considerations of many investors. In this talk, we consider an investor who aims at maximising the expected discounted cash-flow that can be generated by sequentially buying and selling one share of a given asset at fixed transaction costs. We model the underlying asset price by means of a general one-dimensional Ito diffusion X, we solve the resulting stochastic control problem in a closed analytic form, and we completely characterise the optimal strategy. In particular, we show that, if 0 is a natural boundary point of X, e.g., if X is a geometric Brownian motion, then it is never optimal to sequentially buy and sell. On the other hand, we prove that, if 0 is an entrance point of X, e.g., if X is a mean-reverting constant elasticity of variance (CEV) process, then it may be optimal to sequentially buy and sell, depending on the problem data.

A coherent performance measure based on the Sharpe ratio

Mikhail Zhitlukhin

Steklov Mathematical Institute, Russia

The Sharpe ratio is widely used for the evaluation of performance of investment strategies. However it is well-known that it lacks several natural properties of a performance measure, e.g. the monotonicity and the arbitrage consistency. In this talk we present a modification of the Sharpe ratio which does not have these shortcomings.

If X is a random variable representing the excess return of a portfolio, the new measure is defined as the highest Sharpe ratio of all random variables $Y \leq X$ (since the Sharpe ratio is not monotone, a smaller random variable can have a higher Sharpe ratio). In the first part of the talk, it will be shown that this measure satisfies the axiomatic framework for performance measures introduced by Cherny and Madan [1]. The second part will discuss a representation theorem that expresses the value of the measure in terms of the distribution of X in a tractable way. This result allows for the efficient computation of the measure and the solution of the portfolio optimization problem.

[1] A. Cherny, D. Madan (2009). New measures for performance evaluation. Review of Financial Studies, 22(7), 2571-2606.